Weekly Homework 3

Math 485

September 29, 2013

1 Textbook (Stampfli and Goodman)

A. Section 3.1, Page 49: 1,2,3.
B. Section 3.2 Page 51: 1,2,3,4.

2 Additional problems

In all of these problems, Q will denote the risk neutral probability measure. We also take the length of one period, τ to be 1 and the risk free interest rate to be a constant r.

We consider the following formulation for the binominal model:

$$S_k = S_0 X_1 X_2 ... X_k, 1 \leq k \leq n$$

where for $0 < d \leq e^{r\tau} \leq u$, X_i are i.i.d. with distribution

$$\begin{aligned} X_i &= u \text{ with probability } q \\ X_i &= d \text{ with probability } 1-q. \end{aligned}$$

1. (Extra credit - 5 pts) Suppose $r = 0$. Compute

$$E^Q\big((S_5 - S_3)^+ | S_3\big).$$

Interpretation: This is the price for a European call option entered at time $k = 3$ with strike price S_3 and expiration time $n = 5$. Note that S_3 is known at time $k = 3$ so this call option makes sense.

2. (Extra credit - 3 pts) Show that the conditional expectation $E(X|\mathcal{F}_k^S)$ is the best guess of X given $S_0, S_1, S_2, ..., S_k$ in the following sense

$$E\big[(X - E(X|\mathcal{F}_k^S))^2\big] \leq E\big[(X - g(S_0, S_1, S_2, ..., S_k))^2\big], \text{ for all } g.$$

3.Show that in the binomial model, we always have

$$E^Q\big(e^{-r(j-i)}S_j|S_i\big) = S_i.$$

4. Consider the binomial modelwith $n = 10$ and a forward contract on S entered at some time $k, 0 \leq k \leq 9$, strike price K and expiration time $n = 10$.

a) In your own words, explain what $F(6, 10)$ means.

b) Find $F(6, 10)$ using the replicating portfolio approach.

c) Compute $E^Q\big(e^{-4r}(S_{10} - K)|S_6\big)$.

d) Find K such that $E^Q\big(e^{-4r}(S_{10} - K)|S_6\big) = 0$. Compare your answer with the answer in part b.

e) Let V be the value of a forward contract entered at time 0, with strike price $F(0, 10)$ (so that $V_0 = 0$). Compute V_6.

f) Compute $E^Q\Big(e^{-4r}\big(S_{10} - F(0, 10)\big) \mid S_6\Big)$ (Remember $F(0, 10)$ is a known constant). Compare your answer with the answer in part e.

g) Compute $E^Q(e^{-6r}V_6)$. (You should get 0 for the answer here. This is an instance of the rule $E((E(X|Y)) = E(X)$.

5. The Put-Call parity principle says: Holding a long position on a European Call Option and a short position on a European Put Option is the same as holding a long position on a Forward Contract (on the same stock S, with the same expiration date n and strike price K). Suppose S follows the multi-period Binomial model.

a) Express the Put-Call parity principle in terms of $V^{\text{put}}, V^{\text{call}}$ and V^{forward}.

b) Prove the Put-Call parity principle.

Answer: